Joint Solutions of Many Degrees-of-freedom Systems Using Dextrous Workspaces

Sunil K. Agrawal, Ph.D.
Department of Mechanical Engineering
University of Delaware, Newark, DE 19716

Lea Kissner & Mark Yim, Ph.D.
Xerox, Palo Alto Research Center
3333 Coyote Hill Road, Palo Alto, CA 94304

Abstract

In recent years, several studies have focussed on robotic systems with many degrees-of-freedom. Such robots often have stringent joint limits. For motion planning, a key question is to find feasible joint solutions of the system for a given position and orientation of the end-effector. In the presence of joint limits, the solutions are found by searching the joint space using heuristics. In this paper, we propose a simple algorithm to construct the joint solutions for a robot chain with many degrees-of-freedom and joint limits, using dextrous workspaces. The algorithm provides a set of sufficient conditions to guarantee feasible joint solutions in the presence of limits. The procedures are illustrated by theory and experiments on PolyBot, a modular robot developed at Xerox PARC.

1 Introduction

In recent years, for highly versatile applications, robots are being designed with many degrees-of-freedom. Such robots have been referred to as "hyper-redundant" or "modular" in the literature ([3], [4], [2], [8]). These robots usually have a limited range of motion at the joints. Motion planning of such systems requires finding a set of feasible joint angles to place the end-effector in a desired position and orientation. With the built in hyper-redundancy, one would expect that there are many ways to reach a position and orientation. However, in the presence of joint limits, the currently accepted practice is to search for the solutions heuristically. Among these are techniques which rely on the use 'backbone curves' [4], discrete modal summation [2], constrained optimization [5] or more general search, though these tend to be slow and not real-time.

In this paper, our objectives are two-fold: (i) Provide a procedure to identify a subregion of the workspace where a reference point on the end-effector is guaranteed to reach in any orientation, despite the joint limits; (ii) Outline a simple recursive computational algorithm to find the joint solution. The results of this paper are illustrated with *PolyBot* a modular robot system developed at Xerox PARC. The capability to guarantee that a point in the workspace is reachable in any orientation despite joint limits is unique to this work. Consequently, the search procedure changes from a random search to a well informed search, where the existence of the solution is known a priori.

The organization of this paper is as follows: Section 2 outlines the definition of dextrous workspace and its significance in computing the inverse solutions. The dextrous workspace is used to build computational algorithms in Section 3. An implementation of the algorithm on PolyBot is described in Section 4. Issues of parallelization of this algorithm are also described in this section.

2 Dextrous Workspace

The dextrous workspace consists of points in the cartesian workspace where a robot reference point can reach in any orientation of the end-effector. On the other hand, the reachable workspace consists of cartesian points reachable by the reference point in at least one orientation of the end-effector. As one would expect, dextrous workspace is a subset of the reachable workspace. The dextrous workspace is defined for all planar and spatial robot chains ([1],[6],[7]).

In this study, we consider many degrees-of-freedom robot chains with joint limits. We assume that the entire chain consists of N joints. We number the joints and links outwards from the base as 1 to N. The robot end-effector is link N and the reference point is on this link. We assign coordinate frames to the links of the chain according to a standard robot convention. We assume that $\mathcal{F}_0$ is the coordinate frame attached to link 0, i.e. the ground. $\mathcal{F}_i$ is attached to link i of the chain with origin on joint axis i, for $i = 1, ..., N$. We attach an extra frame $\mathcal{F}_{N+1}$ at the reference point on the end-effector parallel to $\mathcal{F}_N$. We denote the origin of a frame $\mathcal{F}_i$ by O_i.

The mathematical problem we are addressing is as follows: "Identify a set of easily verifiable sufficient conditions for a robot chain with joint limits that guarantee that a point in $\mathcal{F}_0$ is reachable by the reference point O_{N+1} in a given orientation of the end-effector. Also, outline a procedure to construct the feasible joint solution".

Theorem 1: *For some $n < N$, partition the partition into n and $N - n$ links on each side of O_{n+1}. Now, identify the following two dextrous workspaces of O_{n+1}: (i) with respect to $\mathcal{F}_0$ using the chain of first n links; (ii) with respect to $\mathcal{F}_{N+1}$ using the last $N-n-1$ links with the coordinate frame $\mathcal{F}_{N+1}$ fixed in the desired position and orientation of the end-effector. If these two dextrous workspaces have common points, O_{N+1} is reachable by the chain in the specified orientation while satisfying all joint limits.*

Proof: Let P be a point that belongs to the two dextrous workspaces of O_{n+1} with respect to $\mathcal{F}_0$ and $\mathcal{F}_{N+1}$. Hence, there exists a feasible configuration of the first n links such that O_{n+1} can be placed at P in any orientation of $\mathcal{F}_n$. Similarly, there also exists a configuration of the last $N - n - 1$ links such that O_{n+1} can be placed at P in any orientation of $\mathcal{F}_{n+1}$. The question now is if there exists a feasible joint configuration of the $n+1$th joint, consistent with its joint limit. A supporting sketch is shown in Fig. 1.

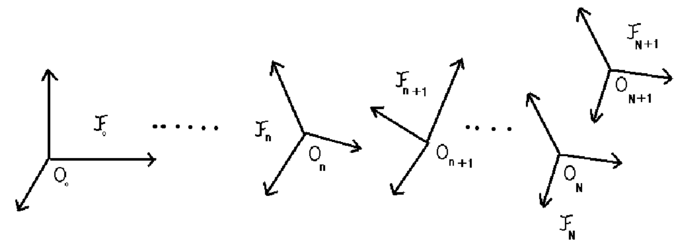

Figure 1: A schematic of a partition of an N link chain into n and $N-n$ links along with the coordinate frames

Due to the property of the dextrous workspace, O_{n+1} can be placed at P using the first n links, with any desired orientation of $\mathcal{F}_n$. The same is true at P for $\mathcal{F}_{n+1}$ using the last $N - n - 1$ links. Hence, given the joint limit of the $n + 1$th joint, one can always choose suitable orientations of $\mathcal{F}_n$ and $\mathcal{F}_{n+1}$ such that the mechanism is assemblable. **End of Proof.**

A larger intersection of common points can be found if instead of the dextrous workspace, the reachable workspace is used for one of the two partitions.

3 Computational Algorithm

Theorem 1 provides a set of sufficient conditions for assemblability of the chain in the presence of joint limits using the property of dextrous workspace. From the point of view of constructing the solution, these results can be applied in several alternative ways: (i) A partition of the chain is made into n and $N - n - 1$ links such that the dextrous workspaces of O_{n+1}, using the two chains, have a common intersection. From the theorem, we are now assured of a set of feasible joint angles which can be computed through search. If required, the two partitions can be further subdivided and the process can continue recursively using the dextrous workspace of the partitions. (ii) The entire chain is partitioned into repeating units, each with a number of joints, such that the dextrous workspace of each unit is well known. A search is then performed over the dextrous workspace of the units to connect the frame $\mathcal{F}_0$ with $\mathcal{F}_{N+1}$. This search places the end-effector in the correct orientation with the reference point at the desired location.

Both these approaches are workable. The first approach requires finding the dextrous workspace of a chain with arbitrary number of joints. If such a characterization is possible, one can very quickly find regions in the workspace which are reachable by the end-effector. Sometimes, it may be difficult to characterize the dextrous workspace of an arbitrary chain. In this case, the second approach can be used with a well defined structure of the dextrous workspace of a unit.

The second approach can be described mathematically as follows: Let N links of the chain be partitioned into l repeating units, with coordinate frames $\mathcal{F}_{ib}$ and $\mathcal{F}_{ie}$ assigned to the beginning and end links of the ith unit, for $i = 1, ..., l$. On each unit, an additional coordinate frame $\mathcal{F}_{ie'}$ is attached to its end link parallel to $\mathcal{F}_{ie}$, but coincident with the origin of $\mathcal{F}_{i+1\ b}$. The frame assignments come from the standard robot conventions of a N link series chain.

Given the coordinate frames $\mathcal{F}_0$ and $\mathcal{F}_{N+1}$, the search problem is to find a sequence of intermediate coordinate frames $\mathcal{F}_{ib}$, $i = 2, ..., l$. This sequence of coordinate frames has to satisfy the property that the origin of $\mathcal{F}_{i+1\ b}$ lies within the dextrous workspace of the unit i with respect to the coordinate frame $\mathcal{F}_{ib}$ and $\mathcal{F}_{N+1}$ is in the desired orientation. Once this is

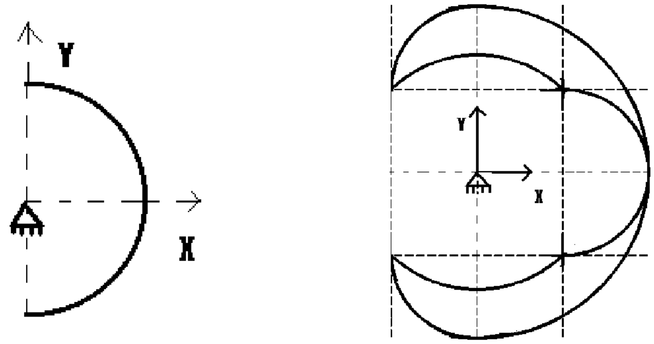

Figure 2: The workspace of a (i) 1-link, (ii) 2-links PolyBot chain with joint limits of of $+\frac{\pi}{2}$ and $-\frac{\pi}{2}$. Each edge of the square denotes the segment length L.

performed, from the property of dextrous workspace, we are guaranteed that feasible joint angles for each unit can be found.

In order to simplify the computations involved in this search, we can impose further properties on the dextrous workspace. For example, we can desire the workspace of each unit to be spherical (or circular for robots moving in a plane). The motivation for such a choice is that intersection of two spheres or circles can be found geometrically or analytically in a simple manner. We would make use of such a property to compute the joint solutions in the following sections.

4 PolyBot: An Example

The concepts in this paper are illustrated with the example of PolyBot, a modular reconfigurable robot. Most configurations of the robot include serial chains. Some instance have had as many as 32 modules in a single chain. In the G1v4 version, pictured in Figure 4, every module has one degree of freedom, batteries and a small 8-bit microprocessor (PIC16F877). Each module also has the ability to connect to one of 4 connection ports. However, we consider an arrangement of PolyBot modules such that all joint axes are aligned normal to a plane. The resulting robot is planar. Each module has a segment length L and a range of motion of $+\frac{\pi}{2}$ and $-\frac{\pi}{2}$ with respect to the neighboring module.

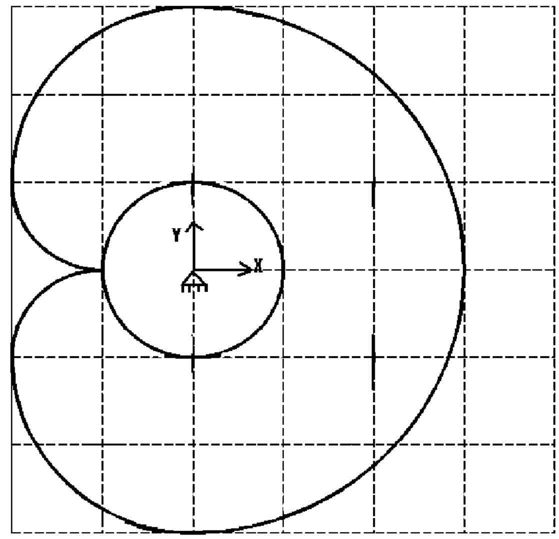

Figure 3: The workspace of a 3-link PolyBot chain with joint limits of of $+\frac{\pi}{2}$ and $-\frac{\pi}{2}$. Each edge of the square denotes the segment length L.

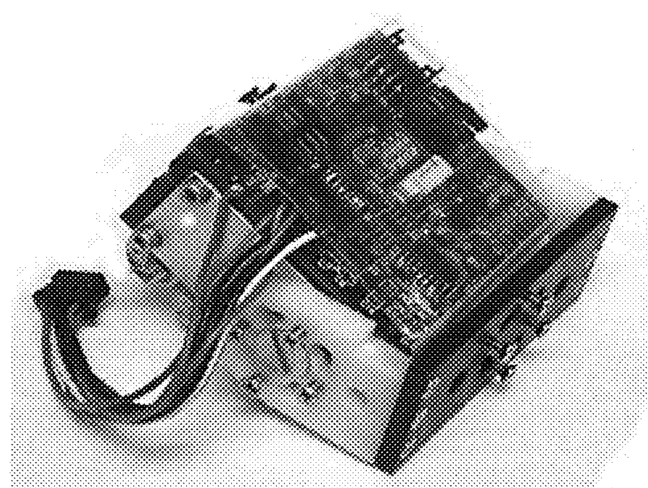

Figure 4: PolyBot G1v4 module with 1 degree of freedom, servo, computer, batteries and 4 connection ports.

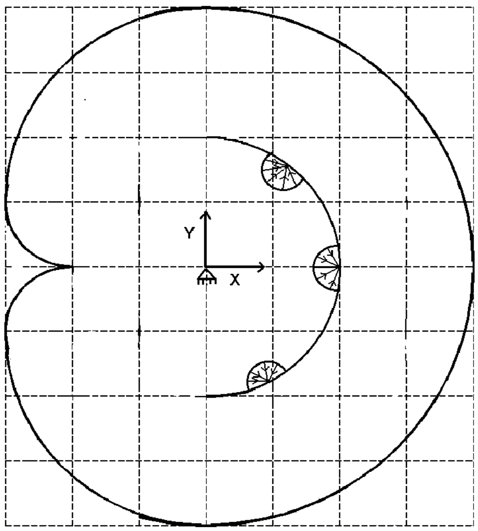

Figure 5: The workspace of a 4-link polyBot chain with joint limits of of $+\frac{\pi}{2}$ and $-\frac{\pi}{2}$. Each edge of the square denotes the segment length L.

4.1 Dextrous Workspace

For 1-module, the workspace of its end point is shown in Fig. 2. As expected, the reachable workspace of the end-point is a semicircle. This workspace does not have any dextrous points. The workspace of the end point for a 2-chain module is also shown in the same figure. This workspace is bounded by six arcs as shown in the figure. Since the range of motion of the first link is $\pm\frac{\pi}{2}$ defined from the positive **X** direction and the range of motion of link 2 is again $\pm\frac{\pi}{2}$ from the nominal pointing direction of link 1, the workspace extends further along the positive **X** direction. This workspace does not have any dextrous points.

The workspace of the end point for a 3-module chain is shown in Fig. 3. It contains a hole and the exterior boundry is made up of five arcs. If P was a dextrous point in this workspace, a circle of radius L around this point must be reachable by the end of a 2-module chain. Since the workspace of Fig. 2(ii) can not accomodate a circle of radius L, it is evident that the workspace of the 3-module chain would not contain a dextrous point either. Similar to the 2-module case, the workspace is more pronounced along the positive **X** direction.

The workspace of the end point for a 4-module chain is shown in Fig. 5. Its exterior boundary has five arcs. A semicircular ring of radius $2L$ is also shown in this figure. A point P on this ring can be reached by the end-point, but only half of all possible orientations. We call this the *half-dextrous ring*. One way to visualize these orientations is to draw a tangent to the semicircle at P. The pointing directions are limited to one side of this tangent line. It can be verified from Fig. 3 that if the end point is on this ring in one of these orientations, the end point of the 3-module chain always lies within its workspace and limits on joint 4 are satisfied. In summary, a 4-module chain does not have fully dextrous points in its workspace. However, it has a semicircular ring of $2L$ with the special property that a point on it can be reached in at least half of all possible orientations. These half rings are used to compute the joint solutions in the next section.

It can be further shown that as the chain length increases, i.e., $n > 4$, where n is the number of modules, the reachable and dextrous workspaces become larger and potentially contain circular and semicircular disks around the origin of $\mathcal{F}_0$. In each case, it can be shown by geometry that the dextrous workspace is bounded by a half-dextrous ring of radius $(n-2)L$.

4.2 Computations

For PolyBot, since the dextrous workspace is fully characterizable for any number of modules, we use the first approach outlined in Section 3 to compute the joint solutions. We assume that the number of modules N of the PolyBot are $8 * 2^i$, where i is some integer. *We choose the goal to reach a specified end-point, while satisfying the joint limits.*

We divide the chain into two equal sub-chains of $\frac{N}{2}$ modules, i.e., the chain is opened up at the $\frac{N}{2}+1$th joint.

The case where $\frac{N}{2}=4$ is treated as a special case, in general $\frac{N}{2} >= 8$. Since the dextrous space is bounded by the half-dextrous ring and there are 8 or more modules in each sub-chain, the dextrous workspace is delineated as the inside of the half-dextrous ring with radius $(\frac{N}{2}-2)L$. Thus, a feasible joint solution exists using the two dextrous workspaces if the total distance between the origins O_0 and O_{N+1} is less than or equal to $2(\frac{N}{2}-2)L$. This process of subdivision of the sub-chains continues in a "tree-like" fashion until one of the following conditions are met: (i) the corresponding dextrous workspaces do not have an intersection; (ii) at least one chain has 4 modules left. The 4 module chain has the half-dextrous half circle which is treated as a dextrous workspace with an extra check to ensure that orientations are chosen in the reachable half

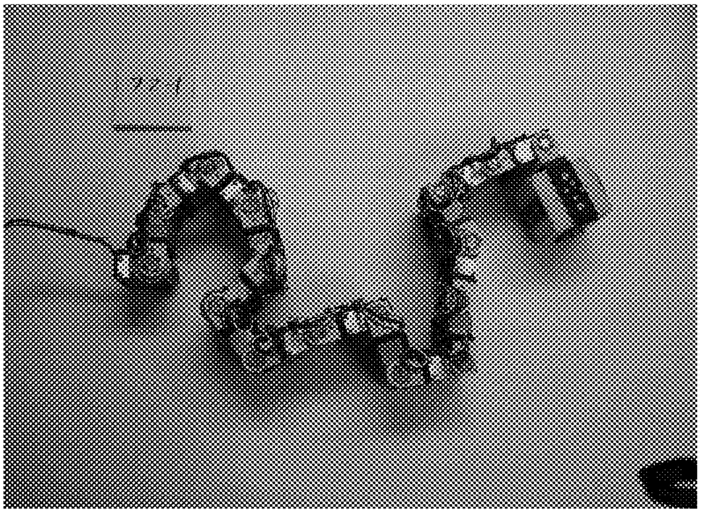

Figure 6: An experiment on a 16-module polyBot, where the end-effector is commanded to a ($x = 7, y = 2, \theta = -1$ rad).

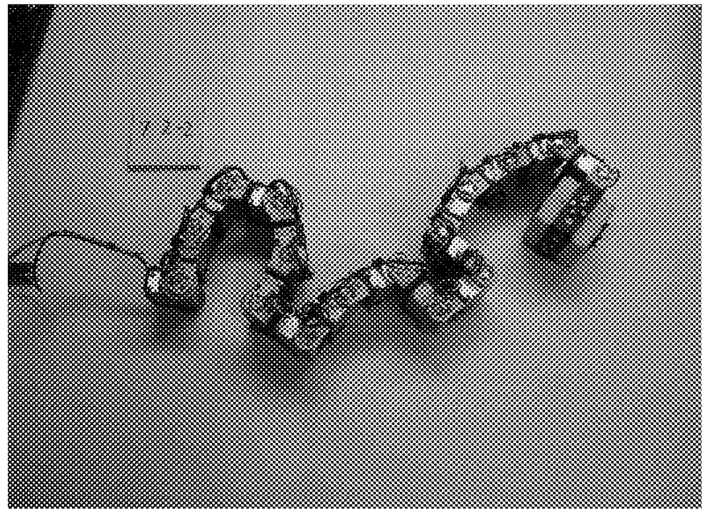

Figure 7: An experiment on a 16-module polyBot, where the end-effector is commanded to a ($x = 7, y = 2, \theta = -2$ rad).

space of orientation.

Regardless of where the process terminates, we are assured after the first step that there is a joint solution within specified limits. This solution can be obtained via a search with the guarantee that the solution will always be found.

One of the salient points of this algorithm is that the search space has a much smaller dimension than N originally posed in the problem. If the procedure terminates with all subchains of length 4, the search reduces to 4 dimensions, even though N could be potentially large. Also, this algorithm is very appropriate for parallelization since one processor can be devoted to solving the joint angles of the lowest subchain in the tree.

4.3 Experiment

The algorithm was verified using 16 PolyBot modules (Generation 1, Version 4) arranged to form a planar chain. Each module has a processor to process position commands. The algorithm was executed on a Pentium which sends commands to the processors over a serial bus. The solutions to arbitrary goal positions were found in the order of seconds.

Since the modules are symmetric and each sub-chain end is in the dextrous workspace, the end point of the whole chain is in a dextrous workspace.

Fig. 6 and 7 show two representative solutions with the end-effector in the same position but different orientation. Fig. 8 shows another example.

Although the algorithm has not yet been implemented in a distributed or parallel fashion, it is expected that it will not be difficult to do so as the hardware with distributed processing exists on the second generation PolyBot modules. Also, self collision is an aspect that is not detailed in the current algorithm using dextrous workspace that is needed to generate realizable joint angles. In the experiment implementation, self collision was checked while searching for a valid solution using Lin-Canny closest features method. In many cases, the collision checking consumes much of the computational resources.

5 Conclusion

A new approach has been proposed in this paper to compute joint solution of a 'many degrees-of-freedom' robot chain with joint limits using the notion of dextrous workspace of a chain. This approach does not rely upon heuristic search but uses sufficient conditions to guarantee the existence of feasible joint solutions. Subregions of the workspace are identified where a reference point is guaranteed to reach in an orientation, despite joint limits. The results of this paper are illustrated with the example of a 16-module robot developed at Xerox PARC. This procedure is highly parallelizable and possesses potential

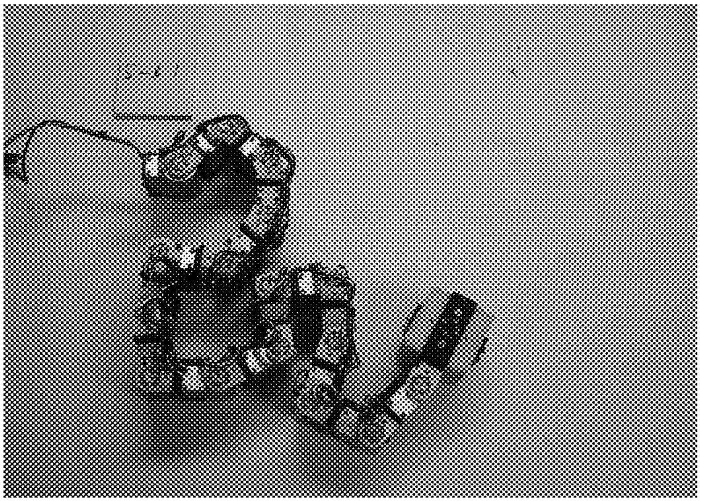

Figure 8: An experiment on a 16-module polyBot, where the end-effector is commanded to a ($x = 5, y = -3, \theta = 1$ rad).

for application on robot systems with many degrees-of-freedom.

Future work includes extending this method to non-planar workspaces (i.e. 6DOF workspaces) implementing a distributed formulation, and incorporating a fast method of detecting collisions.

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