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Telecubes: Mechanical Design of a Module for Self-Reconfigurable Robotics

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Abstract. Telecubes *is a cubic module that has six prismatic degrees of freedom whose sides can expand more than twice its original length and has the ability to magnetically (de)attach to other modules. Many of these modules can be connected together to form a modular self-reconfigurable robot*. *The paper presents the intended functions*, *discusses the physical requirements of the modules and describes two key mechanical components: a compact telescoping linear actuator and a switching permanent magnet device.*

I. Introduction

One of the key ideas in modular self-reconfigurable robotics (MSR) is making ensembles of thousands or more modules that together form a robotic system capable of performing multiple and complex tasks. Many of these systems have been designed simulated and constructed [1,13,14,15,16]. This paper describes the hardware design of *Telecube* which functions as an experimental MSR platform.

Telecubes use telescoping or prismatic degress of freedom to move, attach, and detach from one another (Figure 1). This work extends to three dimensions the robot by Vona and Rus [3] called *Atoms,* that are square (in two dimensions). When a group of *Atoms* are mated they form a *Crystal*. In these systems, cube or square shaped modules rearrange themselves by expanding a face out, detaching or attaching as necessary. The result is similar to the way tiles move in a 8 or 15 -tile sliding puzzles. At the time of this writing two modules have been constructed, another 5-20 are planned.

II. Goals and Methods

The Telecube robotic system is a hardware platform designed for experiments exploring local control methods, distributed sensing and actuation. The application is the forming of arbitrary structural shapes and active surfaces and constructing structures (see Figure 2) [4,5].

The *Telecube* module has two basic mechanical functions: contracting / expanding; and connecting / disconnecting from the faces of neighboring modules. This is accomplished by having each of the six faces attached to independent linear actauators. Each face, called a *connection plate*, has a means to reversibly clamp onto the neighboring module's connection plate, and to transmit power and data to it. The devices which produce the linear extension/contraction and module-to-module clamps are called the *telescoping-tube linear actuator* and the *switching permanent magnet devices*, respectively. These two components carry the most important electromechanical functionality.

Figure 1. CAD solid model of a *Telecube* module in full extension.

The main design requirements that affects the mechanical design of the *Telecubes* are listed in Table 1 and are organized into four areas: 1) primary physical constraints/functions, 2) the docking system functions, and 3) the linear actuator performance, and 4) power and data input/output.

The main design requirements are the shape, size, and number of degress of freedom. A space filling shape is advantageous since it can form solid shapes. This scheme requires an expanded-to-contracted ratio of at least 2:1. Another design goal is a module whose characteristic dimensions are in the low centimeters. This will lead to relatively low cost units that could eventually be manufactured in large quantities. The docking between modules occurs without active guidance to simplify control. To assist in making and breaking connections the insertion and uncoupling forces needed to be very low.

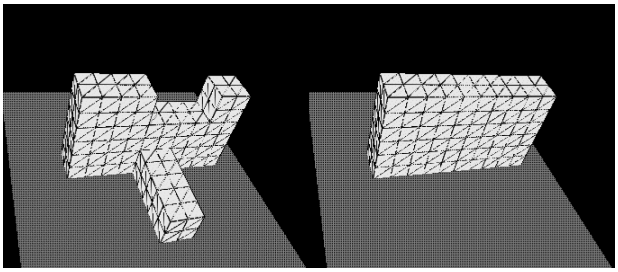

Figure 2. Illustrations of 120 modules reconfiguring into a wall 10x6x2 modules.

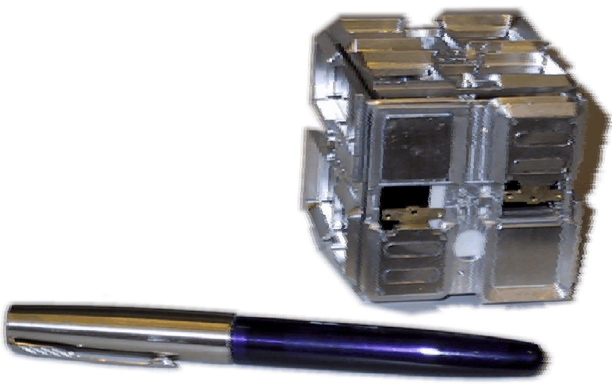

Figure 3. Photograph of prototype module in fully compressed state.

Materials. The *Telecube* module has devices which depend greatly on the characteristics of the materials. The telescoping tube linear actuator has many sliding components which makes static and dynamic friction critical issues. At the same time those elements are structural elements so the choice of material needs to reflect bear, wear and structural requirements. One evaluation metric used to select tube materials, where the section shape and length are already specified, and the area of the tube is to be determined is the following [6]:

$$M = \frac{Y^{1/2}}{\rho} \tag{1}$$

where Y is the flexural modulus and ρ is the specific gravity of the material and the stiffness is likely to be a limiting factor.

Other concerns are the output thrust of the actuation and holding ability to the magnetic devices. Lowering the mass reduces the amount of self-loading on the linear actuators so that a greater percentage of the output thrust can be used for doing work. Lowering the mass also increases the factor of safety on the magnetic holding device.

Table 1. A summary of the major requirements for mechanical design of the *Telecube* module.

Module	
Shape	Cube
Degrees of Freedom	6 prismatic
Size	< 6 cm per size (compact)
Mass	< 300 g
Expansion:contraction	2.2:1
Docking System	
Guidance	Passive
Insertion Force	Zero
Misalignment tolerance	3 mm
Holding Force	5 kg_f
Linear actuator	
Stroke length	≈36-40 mm
Force output	12 N
Input/Output	
Electrical	Electronics, DC motor, GND
Communications	IR Sensor and Transmitter

In addition, proper choice of magnets and magnetic materials are important. The commercially available permanet magnets are stronium ferrite, alnico (aluminum, nickel, cobalt), ceramic, neodymium (neodymium, iron, boron), and samarium cobalt. Magnetic materials also come in a wide range of formulations. Most of the available magnetic metals are iron alloys.

Tube Design. The equation for a deflection (δ_b) of a beam can be estimated with the following equation, from [7].

$$\delta_b = \frac{WL^3}{3EI} \tag{2}$$

where W is the load applied at the end of the tube, L the length of the tube, E the Young's (tensile) modulus, and I the area moment of inertia which for thin walled tubes of thickness t is given by:

$$I = \frac{\pi d^3 t}{8} \tag{3}$$

Lead Screw and Motor Sizing. The main design objectives for the linear actuator are to be compact in both length (≤5 cm) and cross sectional area (≈13 mm) to have a stroke length of 36-40 mm, and thrust capacity of ≈12N. These goals are extrapolated from the overall size goal of the cubic module (≤6 cm/side).

The primary concern for the lead screw design is the amount of torque required to turn the screw given the expected loads. Two conditions are considered: raising and lowering the load when the longitudinal axis of the screw is parallel to the load vector. The necessary torque T to *raise* a load W is [11]

$$T = \frac{Wd_m}{2}\left(\frac{l + \pi\mu d_m}{\pi d_m - \mu l}\right) \tag{4a}$$

The torque required to *lower* the load is found to be

$$T = \frac{Wd_m}{2}\left(\frac{\pi\mu d_m - l}{\pi d_m + \mu l}\right) \quad (4b)$$

where W is the applied force, d_m is the average diameter of the screw, μ is the dynamic friction coefficient, and l is the pitch.

This calculation represents the amount of torque needed to overcome the friction between the nut and screw in lowering the load. In some instances this value may be negative or zero, which means the load may lower itself by causing the nut or screw to spin without any torque input. When the value is above zero, the screw and nut is said to be *self-locking*. Fricton between the screw and nut clearly plays a major role.

Required Nut Length. The length of the internal nut is found in the following equation.

$$L_e = N_e p \quad (5)$$

where N_e is the number of threads necessary to sustain the imposed load without exceeding the limits on bearing and shear stresses and p is the pitch [10]. The length depends on what kind of thread failure is expected: bearing loads, shear loads, or bending loads. Equations for number of threads in each of those cases is given in the Appendix.

Switching Magnet Design Guidelines. A commonly used equation to determine the holding force of a magnet on a surface of magnetic material is [8]:

$$F = 0.577B^2 A \quad (6)$$

where B is the flux density of the magnetic at the surface of the magnet and A is the area of the pole face of the magnet.

III. Results

The Telecube module is designed as an assembly of two major sub-assemblies: the motor core and the six connection plates. The motor core holds all of the gear motors and is the structural center of the module. The six connection plates attach to the ends the of inner tube of the linear actuator. Each contains the switching permanent magnet devices, power interconnects, data transfer devcies, and printed circuit board.

A. Telecube core

The heart of the *Telecube* module is the motor fixturing frame which serves as the part where virtually every other subassembly is attached (see Figure 4 below). Parallelism and orthogonality between the telescoping actuators and by extension the connection plates depend on the precision and stiffness of this core. It is machined out of a block of a semi-crystalline polyethythlene tetraphtalate polymer called Ertalyte® TX (Quadrant Engineered Plastic Products, Reading, PA), which is internally lubricated t provide a low coefficient of friction (μ_d=0.19). The actuator frame has 6 round holes machined to accomodate the telescoping tube linear actuators. Its resulting mass is 29g.

To achieve the size goal the gearmotor powering lead screw is placed inside and concentric to the lead screw itself. The gear motor is a 6 mm diameter, 1.2 watt brushless DC gear motor from Maxon Motor AG of Sachseln, Germany (see Table **2** for a summary the gearmotor's specifications). The motor has integrated hall effect sensors to enable closed-loop control.

The lead screw is made out of 6020-T561 Al alloy [9]. A #1-72 18-8 stainless steel set screw secures the gear output shaft to the lead screw. The motor and its relation to the other components of the telescoping linear actuator are shown in Figure 5.

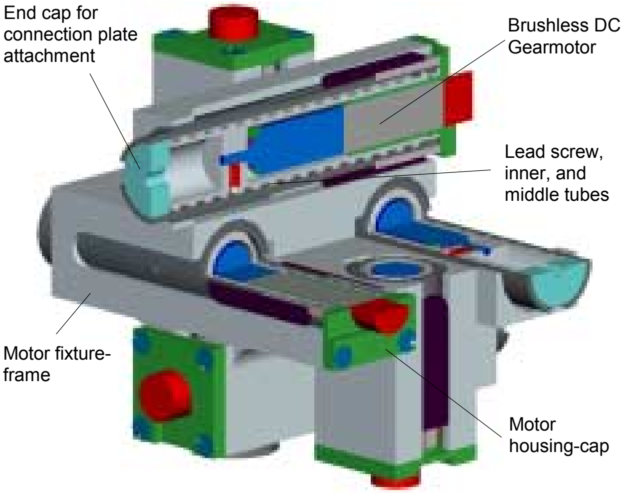

Figure 4. CAD solid model showing the *Telecube's* motor core with six telescoping tube linear actuators installed in a cut-away view.

Table 2. Summary of specifications for the Maxon EC06/GP06 gearmotor.

Gear box: planetary gearhead	
Transmission rate	≈1:221
Number of stages	4
Maximum continuous torque (10s)	60 mN·m
Intermittently permissible torque	80 mN·m
Mass	3.0 g
Motor: Brushless, DC	
Assigned power rating	1.2 W
Nominal voltage	9 V
Torque constant	2.99 mN/mA
Speed/torque gradient	84807 rpm/mN·m
Speed constant	3197 rpm/V
Mass	2.8 g

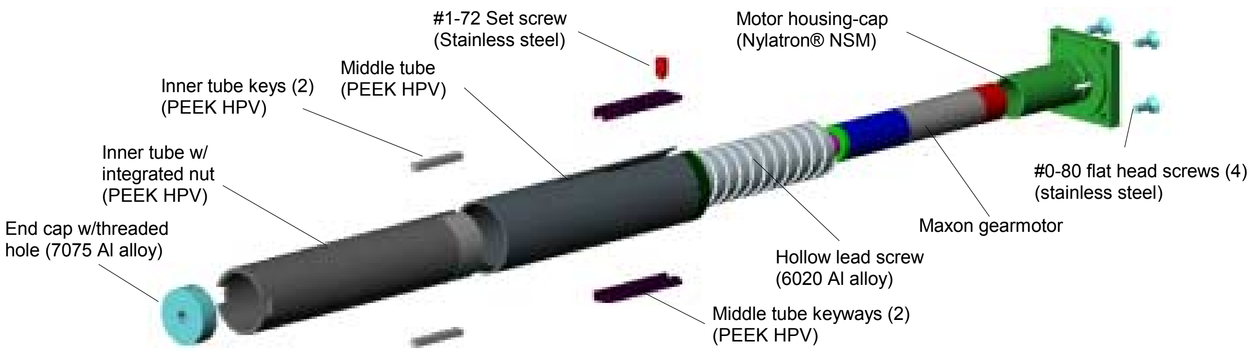

Figure 5. Exploded assembly view of the telescoping tube linear actuator.

The thread is a modified version of the stub Acme thread [10] in that the flank angles are 14.5° but the tooth height is only 0.5 mm (vs. the usual 0.5 × pitch). Each telescoping arm is expected to lift a load of ≈12N which is approximately one module lifting itself along with three other modules. The following assumptions were made: the overall efficiency between the motor output shaft to the lead screw nut thrust to be 40%, the dynamic friction coefficient between the nut and screw is 0.2, the average diameter (ø) of the lead screw is 8.5 mm, and the nominal torque output of the motor is 40 mN·m. Under those conditions, a pitch of 2.7 mm is suitable. To help ensure a low coefficient of friction between the screw and the nut, a 0.0127 mm thick electroless, high-phosphorous nickel coating (Acteron, San Carlos, CA) is applied to the screw. This lead screw is also expected to be self locking.

The gearmotor is fastened by an adhesive to a housing-cap made out of Nylatron® NSM (Quadrant Engineered Plastic Products) that also functions as a sleeve bearing for the lead screw. This nylon has the best combination of wear resistance (wear factor "k" × 10^3 =) and low friction (μ_d=0.18) of all nylons available today. It also has excellent machining characterstics and can be made to tight tolerenaces. The motor housing-cap is designed to snap-fit into the inner diameter of the screw to prevent the gear motor from experiencing tensile axial loads.

The two telescoping tubes of the linear actuator are made out of Ketron® PEEK HPV (Quadrant Engineered Plastic Products) which is a polyetheretherketone filled with carbon fiber, graphite and PTFE. Its tribological properties (low friction, long wear, low mating part wear, high pressure-velocity limits) and its easy machining make it well suited to its role as a structural and bear and wearing component. The smaller of the two tubes for the linear actuator has an integrated nut which helps to reduce the cross sectional area of the linear actuator. Bonded to the tube are two keys for linear guides and an endcap with threaded hole for attachment to the connection plate. The larger of the tubes functions as a linear guide between the inner tube and the actuator fixturing frame. It prevents excessive bending when the inner tube is fully extended from the frame.

The stiffness index of ≈54×10^3 for Ketron® HPV is one the highest for engineered plastics. The expected deflection of the tubes (≈0.6-0.7mm thick×50mm long × 9.5-10mm ø) when fully extended subject to a 29N vertical force at the tip —5× the expected load— is less than 1mm. The resulting square cross sectional area of the telescoping tube actuator is 13.7×13.7mm as measured on the fixturing frame. The linear actuator is designed to extend 36-40mm (or ≈70% of the compact length) at a rate of 8-10mm/sec.

B. Connection plates

The connection plate is the part which defines the outer surface of the *Telecube* module (see Figure 6). It is machined out of Ertalyte®, the unfilled version of Ertalyte® TX. Like the motor fixturing frame, other parts and subassemblies are attached to it: the inner tube of the telescoping linear actuator, the switching permanent magnet device, and the printed circuit board. The printed circuit board and switching permanent magnet devices increase the structural stiffness when attached. The IR sensor and emitter pair and electrical contacts are located in the center of the plate.

The *Telecubes* will not have active guidance during module docking. To guide the last few millimeters of travel before contact, 45° male and female matching surfaces act as chamfers around the magnetic devices. The material also has a low friction coefficient to help in mating. A misalignment of up to 3 mm in any direction parallel to the plane of the plate can be accomodated. The face plate also serves as a kind of foot for the *Telecube* module on horizonal surfaces.

Switching permanent magnet device. The primary means of coupling between modules is by magnetic forces. (see Figure 7a). This solution avoids the issues of pin and hole coupling systems such as binding that can occur when one module is experiencing offset loads when uncoupling. The magnetic system also aids in passive docking.

The device goes from an unlatched to a latched state by shifting the magnetic flux from an internal magnetic path to an external one in which the field of one polarity leaves the pole pieces and enters back into the front magnetic plate. This is accomplished by shifting one set of magnets by one pitch length. In Figure 7b below the magnets are in the "on" state so that magnetic flux would be available for attacting magnetic alloy.

The selection of magnets and magnetic materials is intended to maximize the magnetic holding force. The size of this device is kept to a minimum by using high coercive force magnets such as NdFeB. These magnets are nickel coated discs ≈3.6 mm ø ×1 mm thick (Indigo Instruments, Toronto, Canada) have a BH_{max} rated at 36-38 MGOe. The magnetic metal pieces are all made of an iron alloy called Netic® S3-6 (Magnetic Shield Corp. Bensenville, IL). It has a relatively high magnetic saturation 2.14 T and is relatively easy to machine but is subject to corrosion without a passivating film since it is chemical composition is mostly Fe (≈98%) with small quantities of C, Mn, P, S, and Si.

An estimate of the the flux density of each NdFeB disc is ≈0.35T which translates to a holding force on a magnetic metal of ≈160 g. Therefore, for each switching permanent magnet is estimated to have a holding force of 25N. Since torque to due cantilevered loads are more likely to limit the number of modules that can be held, the switching permanent magnets are positioned at the corners of the plate. With a moment arm length equal to that of the distance between the longitudinal axes of the plate and the magnet device (15 mm), it is estimated that two modules could resist a torque of 0.75 Nm.

The sliding magnet set is pulled to its "off" position, Figure 7a, by a shape memory alloy (SMA) wire (Dynalloy, Inc., Costa Mesa, CA) and returned to its home or "on" position, Figure 7b, by a helical spring . The SMA wire is arranged into an criss-cross pattern which shrinks when heated (see Figure 7e) above the austenite phase transition temperature sliding the magnet array by Δd. The force along the y-axis can be estimated to be

$$F_y = 2F_{SMA} \sin \alpha \tag{7}$$

where α is the angle shown in Figure 7 and F_{SMA} is the force produced by the SMA wire at 4% strain. A 0.254 mm diameter TiNi SMA wire produces ≈2-2.5N of force pulling the magnet by Δd ≈ 3.5 mm switching the flux to an internal path. To return the sliding magnet assembly to the on position, a recovery force of 1.7 N is needed to deform the SMA wire back. A tension spring with a spring rate of 0.47 N/mm is used to overcome the magnetic field resistance as well.

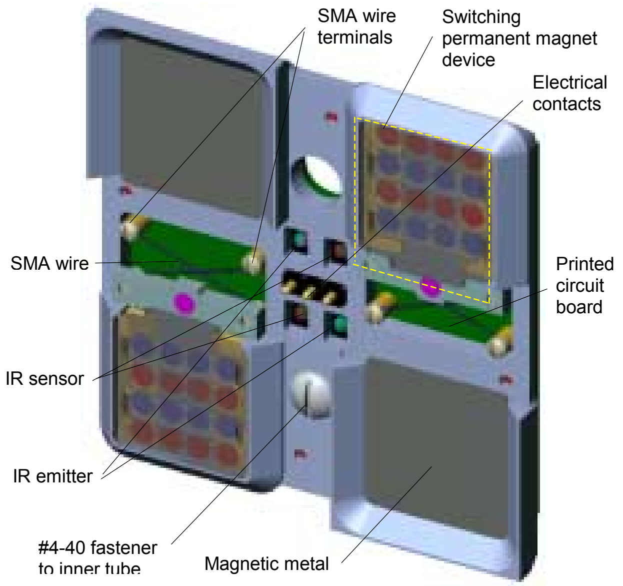

Figure 6. CAD solid model of Connection Plate (right) with switching permanent magnet devices, IR sensors and transmitter, electrical contacts, and printed circuit board.

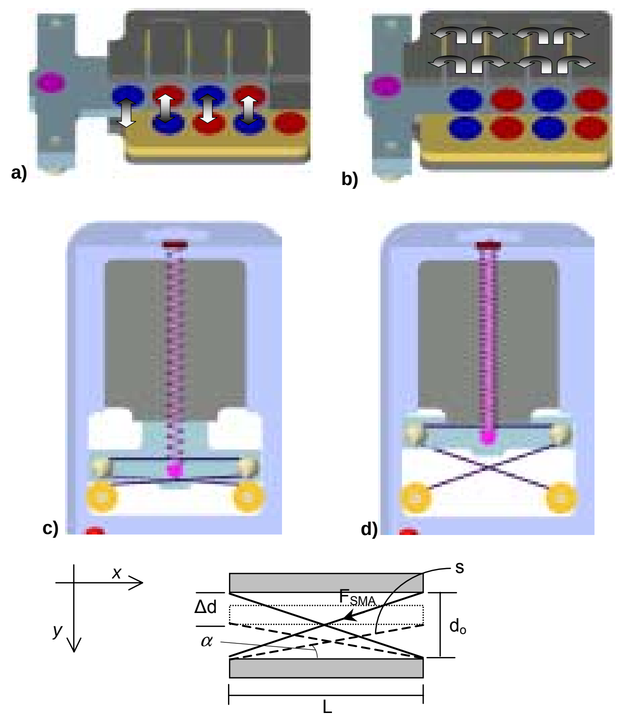

Figure 7. Views of the switching permanent magnet device. a) and c) Device is "on". Red and blue signify magnetic polarity. Arrows show magnetic flux routed internally b) and d) Device is "off." Arrows show magnet flux routed externally e) Schematic of SMA geometry to estimate displacement Δd and force produced.

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Appendix

A. The Telecubes Materials and Their Selected Properties

Table 3. The plastics and their properties used in the mechanical components [12].

	Units	Ertalyte®	Ertalyte® TX	Ketron® PEEK Bearing Grade	Nylatron® NSM	Techtron HPV	Delrin AF Blend Acetal
Description		Semi-crystalline polyester, unfilled	Semi-crystalline polyester w/ solid lubricant	Polyetherether-ketone w/ carbon fiber + PTFE	Nylon 6 w/ solid lubricant	Polyphenylsulfone bearing grade	Teflon filled Acetal
Specific gravity, 25 °C	--	1.41	1.44	1.45	1.15	1.43	1.5
Tensile modulus	GPa	3.7	3.4	5.9	2.8	3.7	3.0
Tensile yield strength	MPa	85	76	75	76	75	55
Elongation at break	%	20	5	5	20	5	15
Flexural modulus	GPa	3	3.4	7.6	3.3	3.7	3.1
Flexural yield strength	MPa	124	96.5	190	110	72	83
Compressive modulus	GPa	2.9	2.8	6.9	2.8	--	2.4
Compressive yield strength	MPa	103	99	34	97	--	110
Dynamic friction coefficient	Dry vs. steel	0.20	0.19	.21	0.18	.16	.15
Limiting pressure velocity	KPa m/sec	44.6	95.5	557	239	270	132
Wear Factor		60	35	100	9	60	85
Water adsorption, sat.	% by wt.	1.0	0.90	0.30	7.0	0.09	1.0

Table 4. Some properties of the magnetic steel and the NdFeB magnets used in Telecube

	Units	Netic S3-6 alloy
Specific gravity, 25 C	--	7.86
Young's (tensile) modulus	GPa	30,000
Tensile strength	MPa	290
Max. saturation induction	G	21,4000
Initial rel. permeability	--	200
Rel permeability at 200 B		500
Maximum rel. permeability	--	4,000
Coercive force	Oe	1.0
Curie temp	°C	770

	Units	NdFeB alloy
Specific gravity, 25 C	--	7.4
Young's (tensile) modulus	GPa	151
Tensile strength	MPa	82.7
Remanence, B_r	kOe	12.1-12.9
Coercive force	kOe	10.8-11.8
Max. energy product	MGOe	36-38
Reverse T coeff of B_r	%/10°	1.23-1.26
Curie temp	°C	310
Max. service temp	°C	80

Table 5. Selected properties of 6020-T562 Al alloy and TiNi shape memory alloy

	Units	6020-T561 Al	TiNi alloy	
			Martensite	Austenite
Specific gravity, 25 °C	--	2.71	6.45	6.45
Hardness, Brinell	500 kg ld/10 mm ball	95	n/a	n/a
Young's (tensile) modulus	GPa	69	28	75
Tensile strength	MPa	262	16.300	
Yield strength	MPa	241	100	560
Elongation at break	%	10	15	15
Thermal conductivity	W/m·K	167	8	18
Coef. of thermal expansion	μm/m·°C	23		
Maximum recovery force	MPa	--	560	
Recommended recovery force	MPa	--	187	
Recommended deform. force	MPa	--	35	
Work output	(joule/g)	--	1	
Energy conversion eff.	%	--	5	
Recommended deform. ratio	%	--	3-5	

B. Equations of the Required Nut Length

Bearing stress. To calculate the bearing area we first observe that the area of the leading flank of one thread is given by:

$$A = \frac{\pi}{4}\frac{D_s^2 - d_n^2}{\cos\alpha_a \cos\lambda}$$

in terms of major screw diameter, minor nut diatmer and leading flank angle.

$$\sigma = \frac{F_n}{A} = \frac{4W\xi}{\pi\left(D_s^2 - d_n^2\right)}\frac{\cos\alpha_1 \cos\lambda}{\cos\theta_n \cos\lambda - \mu_1 \sin\lambda}$$

$$N_e = \frac{4W\xi}{\pi\sigma_b\left(D_s^2 - d_n^2\right)}\frac{\cos\varepsilon_1}{\dfrac{1}{\sqrt{1+\tan^2\alpha_1\cos^2\lambda}} - \mu_1 \tan\lambda}$$

after trigonometric manipulation.

Shear stress. We can only estimate the location of a likely shear surface since the exact location of thread shear failure depends upon both the nut and the screw materials. For this calculation we assume the cylindrical shear surface lies midway between the screw pitch diameter and the nut diameter, then the width b of the thread cut by that surface may be written as

$$b = \frac{p}{2} + \frac{d_p - d_n}{2}\left(\tan\alpha_1 + \tan\alpha_2\right)$$

where α_2 is the following flank angle. From this the minimum number of threads required to avoid shear stress failure is

$$N_{e=}\frac{4W\xi}{3\pi d_n \tau_y b}$$

where τ_y is the shear yeild stress. If an additional factor of safety is desired, use $(d_p$-$d_n)/2$ is used instead of d_p above.

Bearing stress. Bending of the thread causes stress and may be estimated by

$$\sigma = Mc/I\ .$$

We assume that the resultant of the load W acts at the pitch radius and treat the thread to be a wide, helically wound cantilever beam of width πdpNe that is loaded at a distance $(d_p$-$d_s)/2$ from the supported end. The moment M is therefore

$$M = W\left(d_p - d_s\right)/2$$

and the moment of inertia is

$$I = \pi N_e d_p w^3/2$$

where w is the thread width at the minor (root) diameter and corresponds to the depth of the cantilever beam at its built-in (fixed) end. The width w may be written in terms of the thread thickness at the minor (root) radius as

$$w = \frac{p}{2} + \frac{d_p - d_n}{2}\left(\tan\alpha_1 + \tan\alpha_2\right).$$

Manipulation of the equations yields the following equation for N_e

$$N_e = \frac{3W\xi}{\pi w^2 \sigma_y}\left(1 - \frac{d_s}{d_p}\right)$$