# Six Degree of Freedom Sensing For Docking Using IR LED Emitters and Receivers 

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#### Abstract

Six DOF offset sensing between two plates is important for automatic docking mechanisms. This paper presents an easy and inexpensive implementation of such a system using four commercial-off-the-shelf (COTS) infrared (IR) light emitting diode (LED) emitters and two COTS IR receivers on each of two docking plates. The angular intensity distribution of an emitter and the sensitivity distribution of a receiver allow for estimation of the angle and distance between them. Simple experiments have been conducted indicating that such a setup is able to give positional offset in any of 6 degrees of error ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$, pitch, roll, and yaw) within a range. A theoretical framework is also established using least squares minimization. The theoretical framework is general and applies to other configurations of emitter and receiver parts and positioning.


## 1. Introduction and Motivation

Six degree of freedom (DOF) offset sensing between two plates is critical for automatic active docking of self-reconfigurable robot systems such as PolyBot[1] (see Figure 1). Automated active docking requires that the offset errors are measured and then corrected by an automated control system.


Figure 1. PolyBot in spider configuration.
The PolyBot system uses repeated modules all with identical docking mechanisms, or interface plates. Since two interface plates that may dock with each other are identical, they need to have hermaphroditic connection mechanisms. Other systems such as [2][3] may have male and female connection mechanisms, however, the sensing method described in this paper is general and extends to these systems as well.

This paper presents an easy and cheap implementation of 6 DOF sensing system, using four commercial-off-the-shelf (COTS) infrared (IR) light emitting
diode (LED) emitters and two COTS IR receivers on each of the opposing plates. Each of the eight emitters is lit in sequence and an analog reading is taken from both opposing receivers. The angular intensity distribution of an emitter and the sensitivity distribution of a receiver allow for estimation of the angle and distance between them. Simple experiments have been conducted indicating that such a setup is able to give positional offset in any of 6 degrees of error ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$, pitch, roll, and yaw) within a range. A theoretical framework is also established using the least squares minimization. The theoretical framework is general and applies to other configurations of emitter and receiver parts and positioning.

For PolyBot, a first order analysis of the open loop errors indicated that the system can place the interface plates within 30 mm of each other. The mechanical features of the plates are designed to passively mate with up to 3 mm of positional error. The 6 DOF docking sensor system described in this paper is the system developed to close the loop and bring those errors from 30 mm down to 3 mm . IR LEDs and sensors were chosen for this system for their low cost, small size, minimal interface requirements and low processing overhead.

There are a variety of other means for determining the relative position of two objects with 6 DOF, although most are expensive or not suitable for docking. In the Virtual Reality (VR) hardware domain, the use of 6 DOF trackers is a staple. Such systems include linkage based systems, electro-magnetic field based systems[4][5], ultrasonic ranging[6], inertial tracking and vision based methods[7][8]. The ultrasonic and inertial tracking methods have not been extended to 6 DOF in a robust fashion. The vision based methods tend to be computationally intensive and expensive. The electro-magnetic based methods are the most popular for VR however do not work well for self-reconfigurable systems since they are prone to interference by metallic objects and are expensive. Other non-light based positional measurement methods include eddy-current sensing, hall-effect sensor or capacitance based[9] methods. Both of these methods may work however the intimate presence of electric motors may cause too much noise to make the sensing feasible.

Low cost measurement and actuation components may enable applications beyond robotic docking such as: cars that park themselves, jacks on the back of the computer or stereo that move into position as you fumble to plug them in, a gas nozzle that finds the car's tank opening, or robot appliances that automatically dock to recharge, fluid or supply interfaces in your home.

The paper is organized as follows. Section 2 describes the mechanical and electronic design; Section 3 focuses on obtaining the IR intensity model; Section 4 presents the methods of computing 6D offsets; Section 5 discusses experimental results and Section 6 concludes the paper and gives some directions for future work.

## 2. Mechanical and Electronic Design

Figure 2 shows the mechanical design of the plate, where the four small squares at the corners are IR emitters, and two hemispheres along the middle line are IR receivers.

The electrical design ensures that each receiver detects and samples the intensity from each emitter on the opposite plate at a distinct time. In order to do that, each of the eight emitters is lit in sequence and an analog reading is taken from
both opposing receivers. Readings are also taken in between the times when the IR LEDs are emitting to measure the ambient IR. Figure 3 shows the control signals for the first four emitters to be lit. The algorithm decides which side will emit in 'time slot 1 ' and which in 'time slot 2 .' The two sides are synchronized so that the opposing plates measure at the correct time.


Figure 2. Mechanical design of the IR 6D sensing device on a PolyBot faceplate.
At the end of a time period, each of the receivers (total four receivers, two on each plate) will have four readings from their opposing emitters, and four ambient readings totaling 32 measurements. The ambient IR readings are subtracted from the preceding sample to make the system more robust. Hence we end up with 16 pieces of data.


Figure 3. Emitting and receiving sequence.
The current design uses two synchronized Motorola MPC555 PowerPC embedded-controllers to collect the data. In each processor a TPU3 (Time Processing Unit) generates the trigger and emitter control signals. The trigger is fed back into the QADC64 (Queued Analog to Digital Converter) external trigger input
line to obtain the readings from IR detectors. A list of conversions is initially programmed into the A/D queue and a single interrupt is generated at the end of each complete period. The interrupt service routine is responsible for subtracting the ambient measurements and sending the data to the master computation thread. One or both of the MPC555s must send this data over the CANbus network to the master, which is also an MPC555 micro-controller where the algorithm for finding the 6 D position offset is implemented.

## 3. IR Intensity Model

The theory behind this design is based on the fact that the intensity detected by a receiver is a function of the distance and/or angle between the emitter and receiver, i.e., $I=f(e, r, d)$ where $I$ is the intensity reading, $e$ and $r$ are emitter and receiver angles, respectively, and $d$ is the distance between the emitter and the receiver. An accurate model can be obtained by model fitting for given emitters and detectors.

Our model was constructed by decomposing $f$ into three functions, $f_{e}(e), f_{r}(r)$, $f_{d}(d)$ and let $I=A f_{e}(e) f_{r}(r) f_{d}(d)$ where A is a scale factor. We did two separate data collections, one was to fix $r$ to 0 degrees and change $e$ from 0 to 90 in 5 degree increments and $d$ from 0.5 inches to 5 inches in 0.5 inches increments (see Figure 4(a)); and the other was to fix $e$ to 0 and change $r$ from 0 to 90 and $d$ from 0.5 to 5 inches (see Figure 4(b)). The results of this data collection are plotted in Figure 5 (a) and (b), respectively.


Figure 4. (a) Fix receiver angle to 0 degree and change emitter angles and distance. (b) Fix emitter angle to 0 degree and change receiver angle and distance.
We use function $f_{e}=(A-O) e^{-\left(\theta_{e}-\theta_{e}^{0}\right)^{2} / \sigma_{e}{ }^{2}}+O$, where A is 1000 , O is 150 and $\theta_{e}^{0}$ is 20 degrees, to fit the data in a least squares fashion, and obtain the parameter $\sigma_{e}$ as 0.2660 . Similarly, we use function $f_{r}=(A-O) e^{-\left(\theta_{r}-\theta_{r}^{0}\right)^{4} / \sigma_{r}{ }^{4}}+O$ with $\theta_{r}^{0}$ as 40 degrees to fit the receiver data, and obtain the parameter $\sigma_{r}$ as 0.3694 . As a result, we obtain the IR intensity model
$I=(A-O) e^{-\left(\theta_{e}-\theta_{e}^{0}\right)^{2} / \sigma_{e}{ }^{2}} e^{-\left(\theta_{r}-\theta_{r}^{0}\right)^{4} / \sigma_{r}{ }^{4}}+O$. Using this model, we plot the corresponding model data in Figure 6(a) and (b). Compared with Figure 5, the model fits the data relatively well.


Figure 5. Actual intensity changes w.r.t. emitter and receiver angles.


Figure 5. Intensity changes computed by the model w.r.t. emitter and receiver angles.

## 4. Six Dimensional Offset Estimation Methods

For each plate, we attach a frame as shown in Figure 7 (in this case Plate 1 and Plate 2 are facing each other). Given an offset between the two plates, the spatial relationship between each pair of emitter and receiver is determined.


Figure 7. Frames for plates.
Let $d$ be the distance from the receiver to the center of the plate, and $w$ and $h$ be the width and height of the position of the emitters. The coordinate of receiver 1 in its own frame is $\langle 0,0, d\rangle$, and the coordinate of the receiver 2 is $\langle 0,0,-d\rangle$; similarly, the coordinates of emitters $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D are $<0,-w, h\rangle,<0, w, h\rangle,\langle 0$, $w,-h\rangle$ and $\langle 0, w,-h\rangle$, respectively. Let $\langle x, y, \mathrm{z}, \alpha, \beta, \gamma\rangle$ be the offset of the frame of plate 2 with respect to the frame of plate 1 (in the case of two plates facing each other, the offset is $\langle x, 0,0, \pi, 0,0\rangle$ ) and let $T$ be the transform matrix from plate 1 to plate 2 obtained by the offset, and $R$ be the rotation matrix of $T$. The norm of plate 1 is $\langle 1,0,0\rangle$ and the norm of the plate 2 in plate 1 coordinates is $R<1,0,0\rangle$. Let $\left\langle x_{e}\right.$, $\left.y_{e}, z_{e}\right\rangle$ be the coordinate of the emitter in its own frame and $\left\langle x_{r}, y_{r}, z_{r}\right\rangle$ be the coordinate of the receiver of the opposing plate in its own frame. There are two cases:

The emitter is on plate 1 and the receiver is on plate 2: the position of the emitter is $o=\left\langle x_{e}, y_{e}, z_{e}\right\rangle$ and the position of the receiver is $q=T p$ where $p=\left\langle x_{r}, y_{r}\right.$, $\left.z_{r}\right\rangle$ and $q=\left\langle x_{r}^{\prime}, y_{r}^{\prime}, z_{r}^{\prime}\right\rangle$.

The emitter is on plate 2 and the receiver is on plate 1: the position of the receiver is $o=\left\langle x_{r}, y_{r}, z_{r}\right\rangle$, and the position of the emitter is $q=T p$ where $p=\left\langle x_{e}, y_{e}\right.$, $\left.z_{e}\right\rangle$ and $q=\left\langle x_{e}^{\prime}, y_{e}{ }^{\prime}, z_{e}{ }^{\prime}\right\rangle$.

Given two points in space, $o$ and $q$, and the norms of their plates, $n_{o}$ and $n_{q}$, the distance between them is $|q-o|$, the angle at $o$ is $\arccos \left(n_{o} \bullet(q-o)|q-o|\right)$ and the angle at $q$ is $\arccos \left(n_{q} \bullet(o-q) \backslash q-o \mid\right)$. Therefore, the emitter and receiver angles as well as the distance between the receiver and the emitter can be obtained for each of the sixteen pairs of emitters and receivers. Given the IR intensity model we obtained in the previous section, we get a model from each 6D offset between two plates to 16 readings of intensities, i.e., $\mathrm{I}_{\mathrm{i}}=\mathrm{f}_{\mathrm{i}}(\mathrm{x}, \mathrm{y}, \mathrm{z}, \alpha, \beta, \gamma)$ for $\mathrm{I}=1$ to 16 .

### 4.1. Absolute 6 DOF position sensing

Theoretically, the problem of 6D offset estimation becomes a problem of data fitting, i.e., solving $\left\langle x, y, z, \alpha, \beta, \gamma>\right.$ given sixteen data readings. In particular, let $R_{i}$, $i=1 . .16$, be the sixteen readings and let $E$ be an energy function to be minimized,
$E=\frac{1}{2} \sum_{1}^{16}\left(R_{i}-f_{i}(x, y, z, \alpha, \beta, \gamma)\right)^{2}$ which transforms to six equations:
$\frac{\partial E}{\partial p}=\sum_{1}^{16}\left(R_{i}-f_{i}(x, y, z, \alpha, \beta, \gamma)\right) \frac{\partial f_{i}}{\partial p}=0$ where $p$ is $x, y, z, \alpha, \beta$, and $\gamma$.
This set of equations can then be solved using Newton's method. We used Singular Value Decomposition (SVD) for solving linear equations at each Newton step. The use of SVD greatly reduces the risk of reaching a singularity that is very common in problems involving the inverse of matrices. It also achieves a better result in both under- (minimum change) and over-constrained (minimum error) situations.

We applied this method to a set of known offset positions of two plates. To our surprise, the result was not as good as we expected. We examined the problem further and found that the following maybe the major causes:

The particular emitter and receiver pairs we chose were not ideal. First, they have a small range before saturating, and second, the slope in the valid range is too steep, which makes the data extremely sensitive.

The emitters and receivers are sensitive to the mounting position alignment. The resulting variation is very difficult to capture by a simple model. IR ranging using combinations of many plates compounds this problem.

If we fix the two hardware problems in the future, we should be able to get a good absolute position estimation. (This is a good example of good theory that does not necessarily end up with good results in practice).

### 4.2. Relative 6D offset sensing

In the close loop control of the docking process, it is not necessary to have absolute 6D position sensing, as long as (1) it can tell the direction of the offset and (2) it can tell if the offset between the plates are small enough; (1) is used to guide the motion of the plates and (2) is to trigger the latches in the plates to open and close at the right time.

We developed a centering method based on the idea of signal "balancing" when the plates are centered and facing each other. Let $X_{i j}$ represent a reading where $X$ is the emitter ID (A, B, C, or D$), i$ is the receiver ID ( 1 or 2 ) and $j$ is the plate ID (1 or 2), see Figure 7, and let _ represent the case that holds for both plate 1 and 2. When two plates are centered and facing each other, we have a set of equations, e.g., $\mathrm{A} 1_{-}=\mathrm{B} 1_{-}, \mathrm{A} 2_{-}=\mathrm{B} 2_{-}, \mathrm{C} 1_{-}=\mathrm{D} 1_{-}, \mathrm{C} 2_{-}=\mathrm{D} 2_{-}$. In practice, even when the two plates are exactly centered, the equations may not hold because of noise and slight variations when mechanically assembling the plates. The difference, however, can be used as a guideline for a relative offset. For example, ( $\left.\mathrm{A} 1_{-}-\mathrm{B} 1_{-}\right)+\left(\mathrm{A} 2_{-}-\right.$ $\left.\mathrm{B} 2 \_\right)+\left(\mathrm{C} 1_{-}-\mathrm{D} 1_{-}\right)+\left(\mathrm{C} 2_{-}-\mathrm{D} 2 \_\right)$gives offset in Y direction, while ( $\left.\mathrm{A} 1_{-}-\mathrm{C} 2_{-}\right)+\left(\mathrm{A} 2_{-}-\right.$ $\left.\mathrm{C} 1 \_\right)+\left(\mathrm{B} 1 \_-\mathrm{D} 2 \_\right)+\left(\mathrm{B} 2 \_-\mathrm{D} 1 \_\right)$gives relative offset in Z direction. This method has been used to successfully dock two plates in a plane, i.e., a special case with 3D offset.

To follow this path further, we discovered six groups of "balancing" equations, each of which corresponds to an invariant with respect to a subset of 6D offset:

1. Horizontal Group ( $\mathrm{x}, \mathrm{z}, \beta$ invariant): eight equations, four for each plate: $\mathrm{A} 1_{-}=\mathrm{B} 1_{-}, \mathrm{A} 2_{-}=\mathrm{B} 2_{-}, \mathrm{C} 1_{-}=\mathrm{D} 1_{-}, \mathrm{C} 2_{-}=\mathrm{D} 2_{-}$.
2. Vertical Group ( $x, y, \alpha$ invariant): eight equations, four for each plate: $\mathrm{A} 1_{-}=\mathrm{C} 2 \_$, $2_{2}=\mathrm{C} 1_{-}, \mathrm{B} 1_{-}=\mathrm{D} 2_{-}, \mathrm{B} 2_{-}=\mathrm{D} 1 \_$.
3. Diagonal Group ( $\mathrm{x}, \gamma$ invariant): eight equations, four for each plate: $\mathrm{A} 1 \_=\mathrm{D} 2 \_, \mathrm{A} 2 \_=\mathrm{D} 1 \_, \mathrm{B} 1 \_=\mathrm{C} 2 \_, \mathrm{B} 2 \_=\mathrm{C} 1 \_$.
4. Horizontal Cross Group ( $\mathrm{x}, \mathrm{y}, \gamma$ invariant): eight equations between two plates in horizontal direction: $\mathrm{A} 11=\mathrm{B} 12, \mathrm{~A} 21=\mathrm{B} 22, \mathrm{~B} 11=\mathrm{A} 12, \mathrm{~B} 21=\mathrm{A} 22$, C11=D12, C21=D22, D11=C12, D21=C22.
5. Vertical Cross Group ( $x, y, z$ invariant): eight equations between two plates in vertical direction: A11=D22, A21=D12, B11=C22, B21=C12, $\mathrm{C} 11=\mathrm{B} 22, \mathrm{C} 21=\mathrm{B} 12$, D11=A22, D21=A12.
6. Diagonal Cross Group ( $\mathrm{x}, \mathrm{z}, \gamma$ invariant): eight equations between two plates in diagonal direction: $\mathrm{A} 11=\mathrm{C} 22, \mathrm{~A} 21=\mathrm{C} 12, \mathrm{~B} 11=\mathrm{D} 22, \mathrm{~B} 21=\mathrm{D} 12$, $\mathrm{C} 11=\mathrm{A} 22, \mathrm{C} 21=\mathrm{A} 12, \mathrm{D} 11=\mathrm{B} 22, \mathrm{D} 21=\mathrm{B} 12$.
We developed a minimization method that can be used for one or more equations. For example, for equation $A 11=B 11$, we define an energy function $E=\frac{1}{2}(A 11-B 11)^{2}$. Note that this energy function does not have the explicit IR model as the one used for the absolute position sensing. The goal of centering is to move to the direction where the energy function can be minimized. In order to minimize $E$, we calculate $J=\left\langle\frac{\partial E}{\partial x}, \frac{\partial E}{\partial y}, \frac{\partial E}{\partial z}, \frac{\partial E}{\partial \alpha}, \frac{\partial E}{\partial \beta}, \frac{\partial E}{\partial \gamma}\right\rangle$ and $H$ where $\frac{\partial E}{\partial p}=(A 11-B 11)\left(\frac{\partial A 11}{\partial p}-\frac{\partial B 11}{\partial p}\right), p$ is $x, y, z, \alpha, \beta, \gamma$ and $H$ is a $6 \times 6$ matrix with $H_{p q}=\frac{\partial^{2} E}{\partial p \partial q} \approx\left(\frac{\partial A 11}{\partial p}-\frac{\partial B 11}{\partial q}\right)\left(\frac{\partial A 11}{\partial p}-\frac{\partial B 11}{\partial q}\right)$, in which, $p, q$ are $x, y, z, \alpha, \beta$ or $\gamma$. By using SVD to solve the linear equation $H \Delta p+J=0$ where $\Delta p=\langle\Delta x, \Delta y, \Delta z, \Delta \alpha, \Delta \beta$, $\Delta \gamma>$, we obtain the direction of the offset movement to minimize the energy function defined by the equation.

Given a set of equations $1 . . k$, we define the energy function as the sum of the energy functions of each equation $E=\sum_{1}^{k} E_{i}$. Therefore $\frac{\partial E}{\partial p}=\sum_{1}^{k} \frac{\partial E_{i}}{\partial p}$ and $H_{p q}=\frac{\partial^{2} E}{\partial p \partial q}=\sum_{1}^{k} \frac{\partial^{2} E_{i}}{\partial p \partial q}$ in which, $p, q$ are $x, y, z, \alpha, \beta$ or $\gamma$. By solving the linear equation $H \Delta p+J=0$ where $\Delta p=\langle\Delta x, \Delta y, \Delta z, \Delta \alpha, \Delta \beta, \Delta \gamma\rangle$, we obtain the direction of the offset movement to minimize the energy function defined by the set of equations.

The groups of equations we defined can be used to calculate the subset of offsets that are not invariant of the equations. For example, Group 1 equations can be used to calculate $y, \alpha$ and $\gamma$, Group 2 equations can be used to calculate $z, \beta$ and $\gamma$, Group 5 equations can be used to calculate $\alpha, \beta$ and $\gamma$. Also we can combine all the groups and calculate $y, z, \alpha, \beta$ and $\gamma$. To calculate x , we use energy function
$E=\frac{1}{2} \sum_{\substack{i=1,2 \\ j=1,2}}\left(A i j^{2}+B i j^{2}+C i j^{2}+D i j^{2}\right)$, based on the fact that all the readings go to minimum when x approaches 0 in centered position. For simplicity, assuming the plates are centered, we have

$$
\Delta x=-\frac{\partial E}{\partial x} / \frac{\partial^{2} E}{\partial^{2} x}, \text { where } \frac{\partial E}{\partial x}=\sum_{\substack{i=1,2 \\ j=1,2}}\left(A i j \frac{\partial A i j}{\partial x}+B i j \frac{\partial B i j}{\partial x}+C i j \frac{\partial C i j}{\partial x}+D i j \frac{\partial D i j}{\partial x}\right)
$$

$$
\text { and } \frac{\partial^{2} E}{\partial^{2} x} \approx \sum_{\substack{i=1,2 \\ j=1,2}}\left(\left(\frac{\partial A i j}{\partial x}\right)^{2}+\left(\frac{\partial B i j}{\partial x}\right)^{2}+\left(\frac{\partial C i j}{\partial x}\right)^{2}+\left(\frac{\partial D i j}{\partial x}\right)^{2}\right) .
$$

## 5. Experimental Results

We developed an experimental setup for measuring 6D offset. The setup includes two PolyBot modules each of which has an IR plate and one module for calculating the offset. The three modules communicate via CANbus and each contains an MPC555. The outputs of the 6D offset are sent from the computing MPC555 to CANalyzer, which is a CAN interface program running on a PC. The experiments are done basically by fixing one IR module and moving the other IR module in space (see Figure 9).


Figure 9. CANalyzer trace graphics window.
We first experimented with each set of equations individually to find the sensitivity of each of the six dimensions offset with respect to the set of equations. We found that using individual groups for calculating individual offsets, in this case Group 1 for $y$, Group 2 for $\gamma$, Group 4 for z, Group 5 for $\alpha$ and Group 6 for $\gamma$ works better than using all the equations to calculate all the offsets at once. Offset $x$ will only be calculated if other offsets are small.

When the plates are very close, using the current emitter-receiver placement, all the readings tend to approach zero that results in loss of sensitivity. This is an implementation limitation and not a limitation of the method.

## 6. Conclusions and Future Work

We have presented an integrated system, with mechanical-electrical design and embedded software for obtaining a six degrees of freedom offset between two opposing plates for the purpose of docking. The system is simple and cheap, using eight IR emitters and four IR receivers. The software is general and robust using minimization techniques. The same algorithm used for 6 DOF offset estimation is also used for inverse kinematics, similar to [10].

For future work, we plan to improve the IR curve to reduce the saturation range and extend sensitivity in the unsaturated range, so that 6 D absolute positioning may be obtained. We also plan to rearrange the positions of IR emitters and receivers so that the receivers still measure signals when the two plates are docked without loss of sensitivity.

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